

Physics 215C QFT Spring 2026

Assignment 1

Due 11:59pm Monday, April 6, 2026

1. An application of effective field theory in quantum mechanics.

Consider a model of two canonical quantum variables ($[\mathbf{x}, \mathbf{p}_x] = \mathbf{i} = [\mathbf{y}, \mathbf{p}_y]$, $0 = [\mathbf{x}, \mathbf{p}_y] = [\mathbf{x}, \mathbf{y}]$, etc) with Hamiltonian

$$\mathbf{H} = \mathbf{p}_x^2 + \mathbf{p}_y^2 + \lambda \mathbf{x}^2 \mathbf{y}^2.$$

(This is similar to the degenerate limit of the model we studied in lecture with two QM variables where both natural frequencies are taken to zero.)

- (a) Based on a semiclassical analysis, would you think that the spectrum is discrete or continuous?
- (b) Study large, fixed x near $y = 0$. We will treat x as the slow (= low-energy) variable, while y gets a large restoring force from the background x value. Solve the y dynamics, and find the groundstate energy as a function of x :

$$V_{\text{eff}}(x) = E_{\text{g.s. of } y}(x).$$

- (c) Presumably you did the previous part using your knowledge of the spectrum of the harmonic oscillator Hamiltonian. Redo the previous part using path integral methods.
- (d) The result for $V_{\text{eff}}(x)$ is not analytic in x at $x = 0$. Why did that happen?
- (e) Is the spectrum of the resulting 1d model with

$$\mathbf{H}_{\text{eff}} = \mathbf{p}_x^2 + V_{\text{eff}}(\mathbf{x})$$

discrete? Is this description valid in the regime that matters for the semiclassical analysis?

[Bonus: determine the spectrum of $\mathbf{H}_{\text{eff}}$.]

2. Entanglement and EFT. [Bonus problem]

In a given state ρ_{AB} , one measure of the entanglement between two parts of a bipartite Hilbert space $\mathcal{H} = A \otimes B$ is the entanglement entropy, the von Neumann entropy $S_A \equiv -\text{tr}_A \rho_A \log \rho_A$ of the reduced density matrix of one of the parts,

$\rho_A = \text{tr}_B \rho_{AB}$. When the whole system is in a pure state $\rho_{AB} = |\psi\rangle\langle\psi|$, this is a useful measure of their entanglement.

Consider the problem of two oscillators we discussed in lecture as a parable about integrating out high-energy modes. Compute the entanglement entropy S_q of the light mode q in the groundstate of the combined system, by whatever means necessary.

If $S_q \neq 0$, it means that q does not have its own wavefunction, only a probability distribution on wavefunctions. Show that S_q is very small in the limit $\omega_0 \ll \Omega$, so that it is not a terrible idea to approximate the low-energy physics of q as being described by a pure state.

3. **Emergence of the Dirac equation.** (Some words of motivation: we're going to study some interesting phenomena involving the field theory of a Dirac fermion; some of them involve questions about regularizing this field theory. So it will be instructive to have in mind a very concrete and physical regularization of this field theory.)

Consider a chain of free fermions with

$$H = -t \sum_n c_n^\dagger c_{n+1} + h.c.$$

Show that the low-energy excitations at a generic value of the filling are described by the massless Dirac lagrangian in 1+1 dimensions. [Hint: the problem has (discrete) translation-invariance and is linear.]

Find an explicit choice of 1 + 1-d gamma matrices that matches the answer from the lattice model. Show that the right-movers are right-handed $\gamma^5 \equiv \gamma^0 \gamma^1 = 1$ and the left-movers are left-handed.