University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 220 Symmetries Fall 2024
Assignment 9

Due 11:59pm, Thursday November 28, 2024

This assignment is brief because I posted it late.

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1. Prove that any generator in any finite-dimensional representation of a compact, semisimple Lie algebra is traceless. (This fact has important implications for grand unification of forces.)

2. **Some subgroups of** $\mathsf{SU}(3)$**.**

   (a) Consider the subalgebra of $\mathsf{su}(3)$ generated by T_1, T_2, T_3 (where $T_A \equiv \lambda_A/2$ and λ_A are the Gell-Mann matrices). What algebra is it? How does the $\mathbf{3}$ of $\mathsf{SU}(3)$ decompose into representations of this algebra? What about the adjoint of $\mathsf{SU}(3)$?

   (b) Consider the subalgebra of $\mathsf{su}(3)$ generated by T_2, T_5, T_7. What algebra is it? How does the $\mathbf{3}$ of $\mathsf{SU}(3)$ decompose into representations of this algebra? What about the adjoint of $\mathsf{SU}(3)$?

   Hint: the adjoint of $\mathsf{SU}(3)$ is $\mathbf{3} \otimes \bar{\mathbf{3}}$ minus the singlet. So if you know how the $\mathbf{3}$ transforms under a subgroup, you can figure out how the adjoint transforms, too.